Remember the Past and Forget Thresholds

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Task Allocation

(a) Foraging

(b) Brood care

(c) Farming aphids

(d) Cultivating fungi

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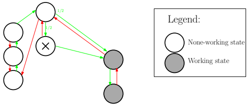
- Response thresholds: Different ants have different thresholds for each task, i.e., they perform the task when the stimulus is larger than the threshold.
- State machines: Ants have different states and transition from one state to another based on the interaction with the environment.

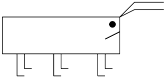


Response Thresholds

- Wilson (76): Observed that ants (*P. dentata*) split into two castes (majors and minors) where each caste performs a different set of tasks.
- Bonabeau et al. (96), study a theoretical version of this. One tasks and different thresholds θ_1, θ_2 for the two castes. Distribution of majors and minors agrees with experimental data of Wilson.
- Page et al. (98) use a slightly different model and assumes that the thresholds are drawn from different distribution.
- Evolution of thresholds has also been studied.

Let’s model this!

- We’ll model ants and tasks!



An accurate depiction of the *Squarus Formica*

Model - Tasks

- We have k tasks.
- demand: Each task i $\mathrm{demand}^{(j)}$, i.e., number of ants required to work on task j.
- load: $\mathrm{load}_j^{(t)}$ is the number of ants working on it at time t.
- deficit: $\mathrm{deficit}_j^{(t)}$ is the $\mathrm{demand}^{(j)} - \mathrm{load}_j^{(t)}$ at time t.

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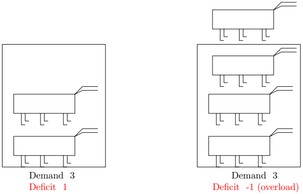
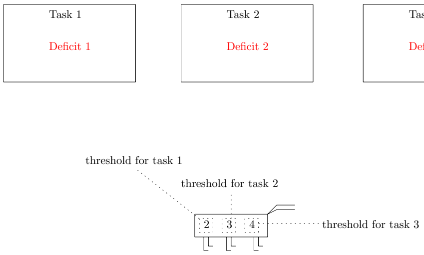


Two tasks, one underloaded and one overloaded.

Model - Ants

- Say we have n ants.
- Assume we have synchronous rounds
- For each task j, each ant i has a (joining) threshold $j_i^{(j)}$
- If $\text{deficit}_j^{(t)} \geq j_i^{(j)}$ and ant is idle, then the ant i joins the task j.



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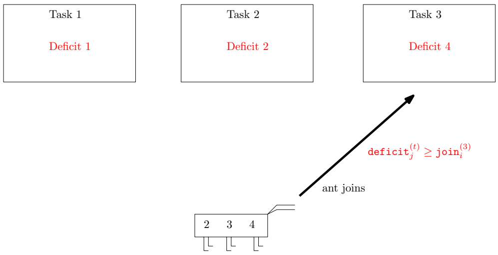

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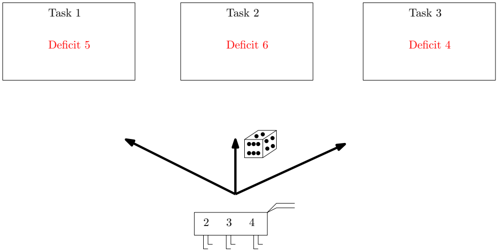
- In case there are several tasks exceeding the joining threshold, the ant picks one *uniformly at random*.



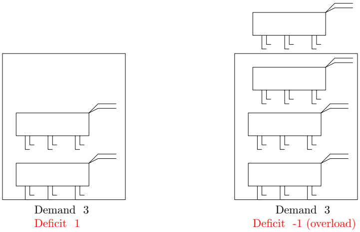
All deficits are above the threshold. The ants joins a task chosen uniformly at random.

When do ants leave?

- We assume all ants have the same leaving threshold
- If the overload exceeds this threshold, all ants will leave.

What is possible?

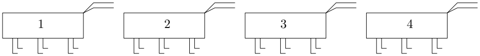
- Quality measure Q for task allocation: Sum of absolute deficits.
  For example: $Q = |1| + |-1| = 2$



- 1 Task: Perfect allocation possible ($Q = 0$)
- 2 Tasks: Perfect allocation possible ($Q = 0$)
- 3 Tasks: ??? (our main result)
- I'll talk about all three

1 Task

- Give numbers from 1 to n to the ant.
- Ant i has a joining threshold of i.
- The leaving threshold (for all ants) is arbitrary



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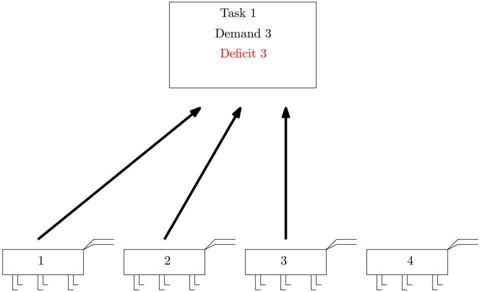

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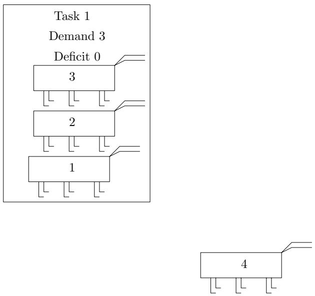

2 Tasks

- Give numbers from 1 to n to the ant.
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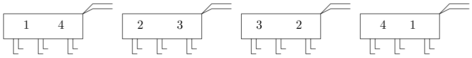
- The leaving threshold (for all ants) is some arbitrary positive value



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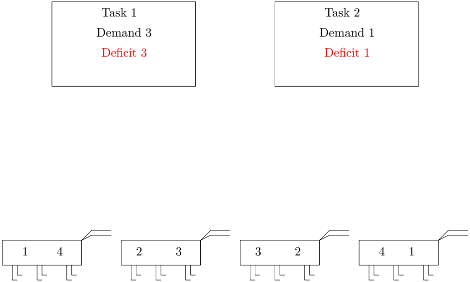

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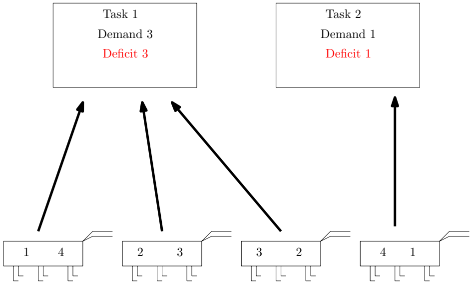

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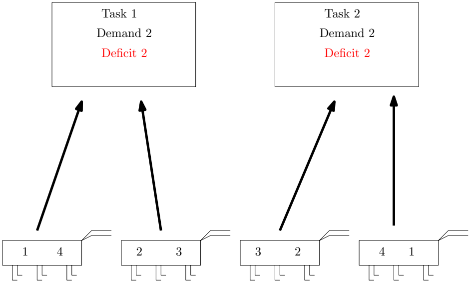

Demands changed from $(3, 1)$ to $(2, 2)$

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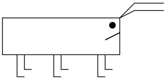
Note: We always assume that the sum of demands are at most n; otherwise it's impossible anyway ...

3 Tasks - Our Theorem

- In general, it is impossible to reach a perfect allocation ($Q = 0$)!
- In particular, for any setting of thresholds, there is a demand vector where the sum of absolute values is linear in n.



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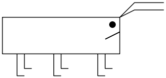
- In general, it is impossible to reach a perfect allocation ($Q = 0$)!
- In particular, for any setting of thresholds, there is a demand vector where the sum of absolute values is linear in n.
- Disclaimer: This does not mean that ants don't do it, but if they do, it's highly inefficient.
- Note, state-machines are capable of reaching perfect allocation ($Q = 0$) in even harsher settings



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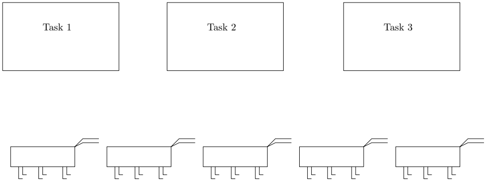

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- Say we have n with arbitrary joining thresholds.
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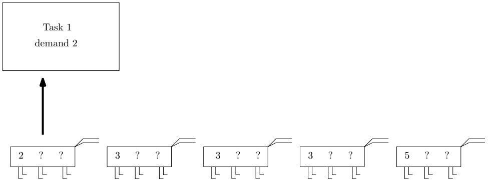
- Consider the number of ants n_1 that would join task n_1 if it had a demand of 2.
- Case $n_1 < 2$: then too few ants will join



Deficit will always remain 1 :(

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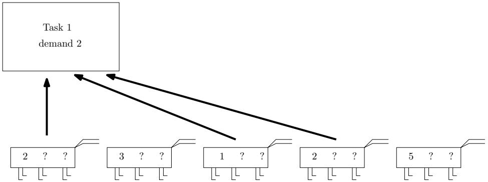
- If $n_1 > 2$ then too many ants join



Two cases, both bad: Deficit will always remain -1 or endless oscillations

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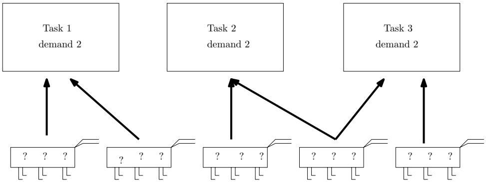

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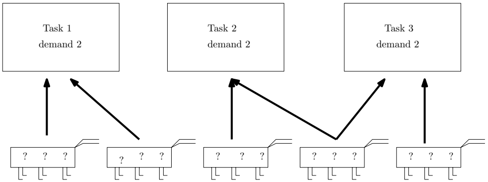
- Incoming edges of the task should be 6
- Thus (at least) one ant would join two different tasks.



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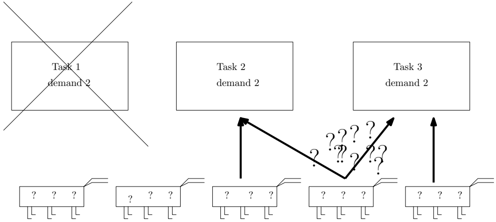

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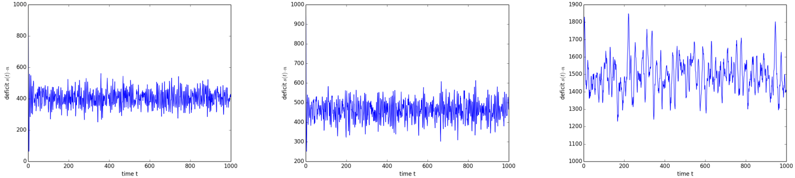
- Why hasn't this been noticed before?
- First, many papers only study two tasks ...
- Second, most of the papers don't analyze any quality measure. If they do, it's often the proportion of minors and majors in the nest.

Boneabeau Model - Huge oscillations

The following figure shows our simulation of this protocol with different fractions of cast 1 (n_1/n). In all different settings of n_1/n we see large oscillations of the deficit. The oscillations appear to be linear in the number of ants.

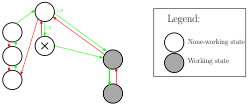


$n_1/n = 0.2$ $\quad$ $n_1/n = 0.3$ $\quad$ $n_1/n = 0.7$

Figure: Parameters are $N = 1000$, $\theta_1 = 8$, $\theta_2 = 1$, $\delta = 1$,$\alpha = 3$ as suggested by Bonabeau et al. (96) and Bonabeau et al. (98).

State Machines

- The performance of state-machines can be much better $Q \leq 1$
- In every time step an ant is in a state (marked with an **x**) and receives some feedback.



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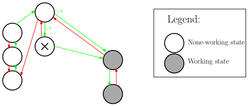
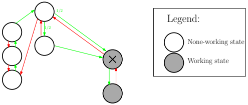


- Based on that feedback (e.g. $\rightarrow$) the ant transitions to the next state

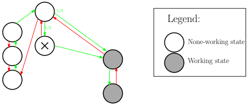


State Machines

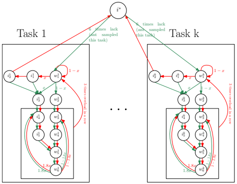
- Meyer et al. showed that
  - threshold model (for bumblebees) does not match experimental results.
  - They suggest a time-resolved model (which can be seen as a simple state machine).

State Machines

- Cornejo et al. proves that for binary feedback (overload/underload of task) achieves a 'perfect allocation' ($Q \leq 1$)



- Dornhaus et al. (BDA last year) show that this still works ($Q << n$) even when the feedback (overload/underload of task) is noisy



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- Indicating that there is no need for permanently different
- Using a tiny bit of memory (state machines) allows ants to ‘differentiate’ temporarily
- Of course you could combine both models, but is there an advantage?

Thank You

Questions?