Convex Hull Formation for Programmable Matter

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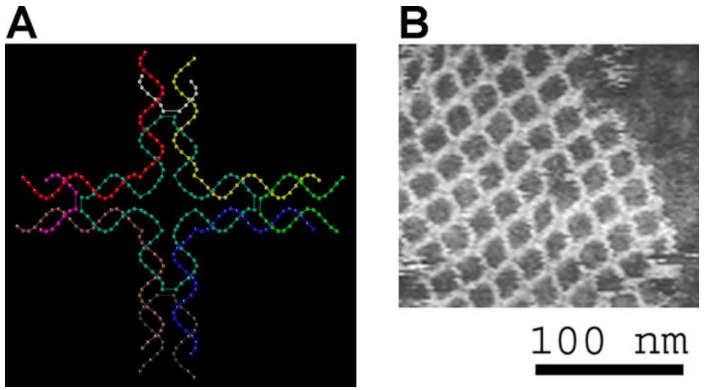
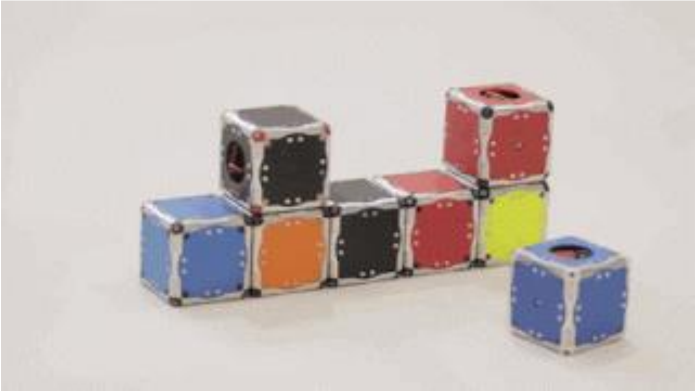
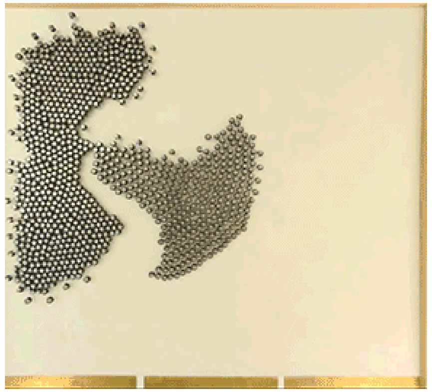
Current Programmable Matter

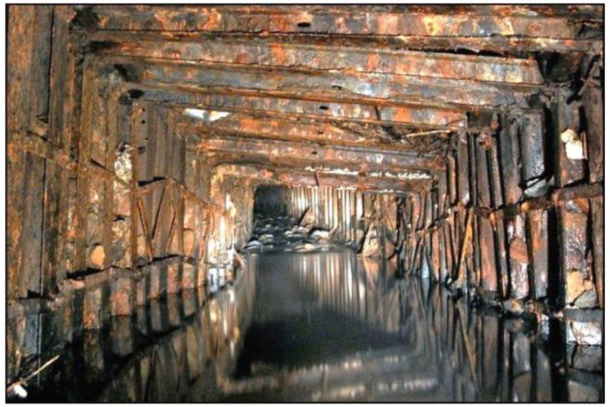


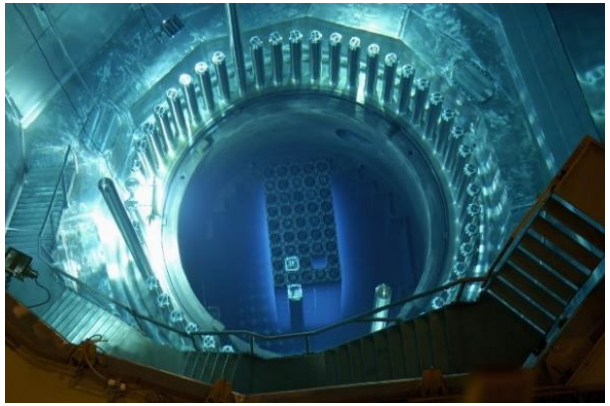


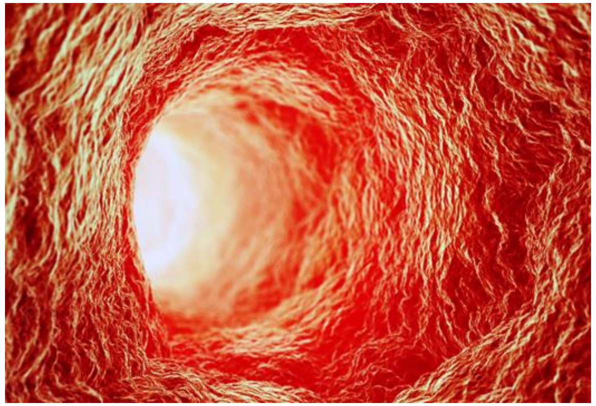


[1] RGR 2013: "M-blocks: Momentum driven, magnetic modular robots" [2] RCN 2014: "Programmable self-assembly in a thousand-robot swarm"

Inspirations & Applications



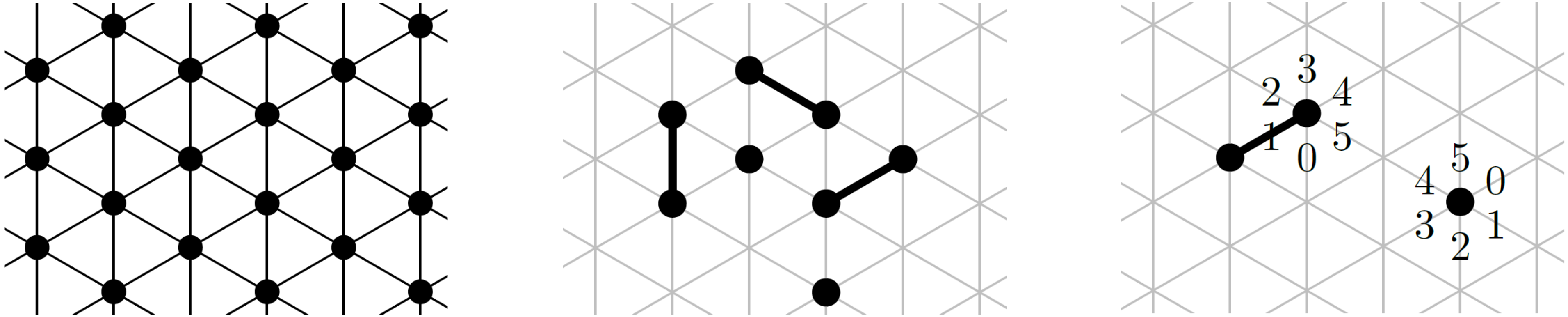




The Amoebot Model

Particles move by *expanding* and *contracting,* and are:

- Anonymous (no unique identifiers)
- Without global orientation or compass (no shared sense of "north")
- Limited in memory (constant size)
- Activated asynchronously

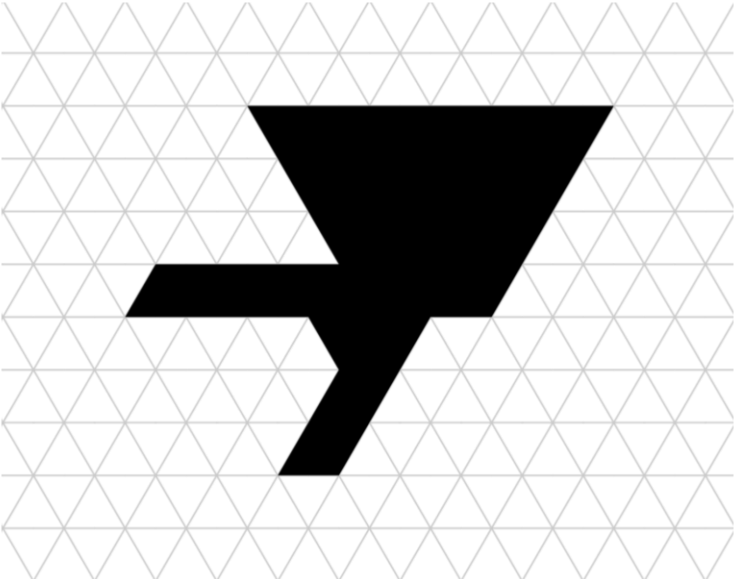


Our Past Work

- Leader Election [DNA21, ALGOSENSORS '17]
- Shape Formation [NANOCOM '15, SPAA '16]
- Object Coating [*Theoretical Computer Science, Natural Computing*]
- Full list of publications can be found at: sops.engineering.asu.edu/publications-press/.

Convex Hull: Definitions

- We begin with an object *O*, which is a connected set of nodes in our graph $G = (V, E)$.



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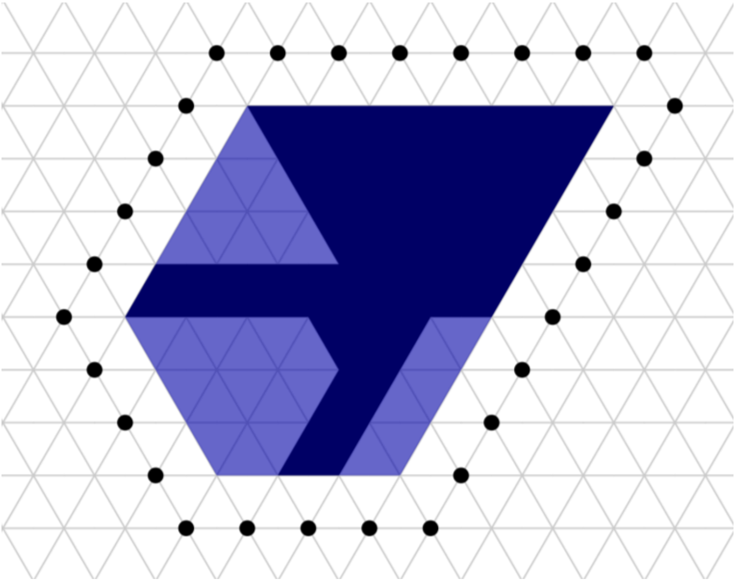
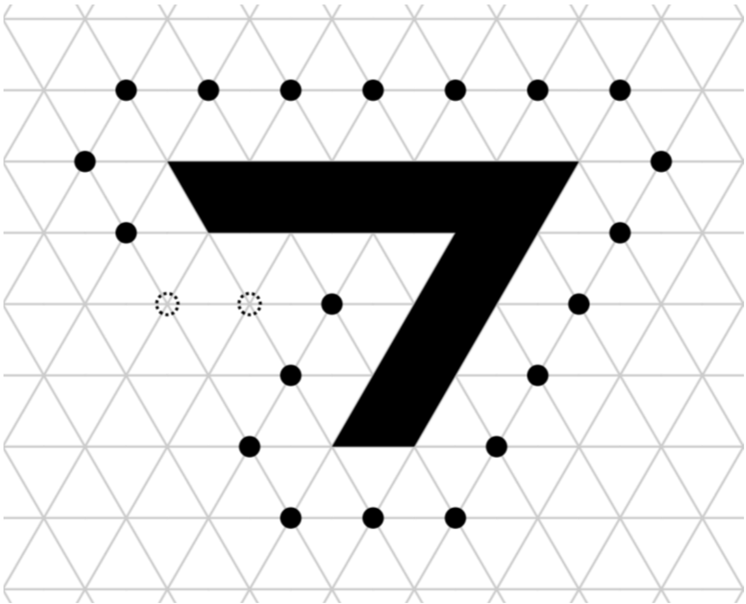
- Let *O** be the minimal convex set of nodes containing *O*.
- The *convex hull* of *O*, denoted *C(O)*, is the set of nodes in *V* \ *O** adjacent to some node(s) of *O**. (Essentially the "external boundary" of *O**).



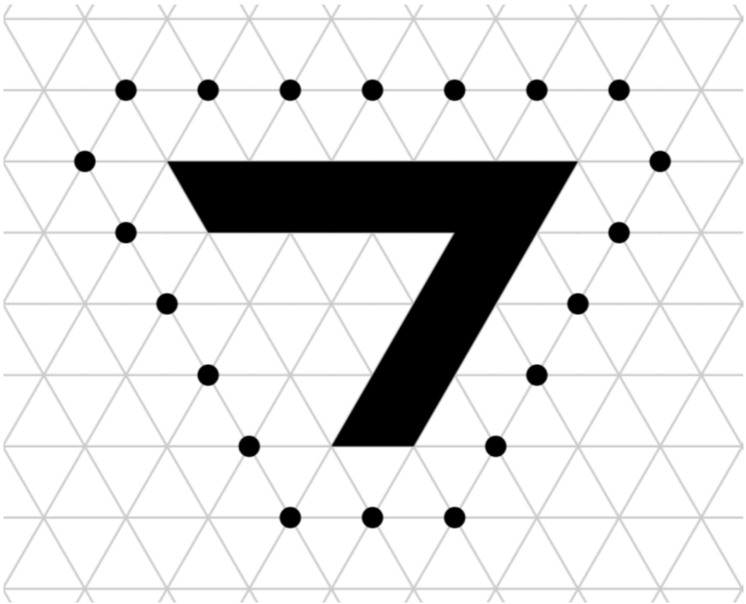
Why Convex Hulls?

- Interesting problem in computational geometry, especially in distributed settings.
- Can be viewed as a relaxation of object coating.

Incomplete Coating



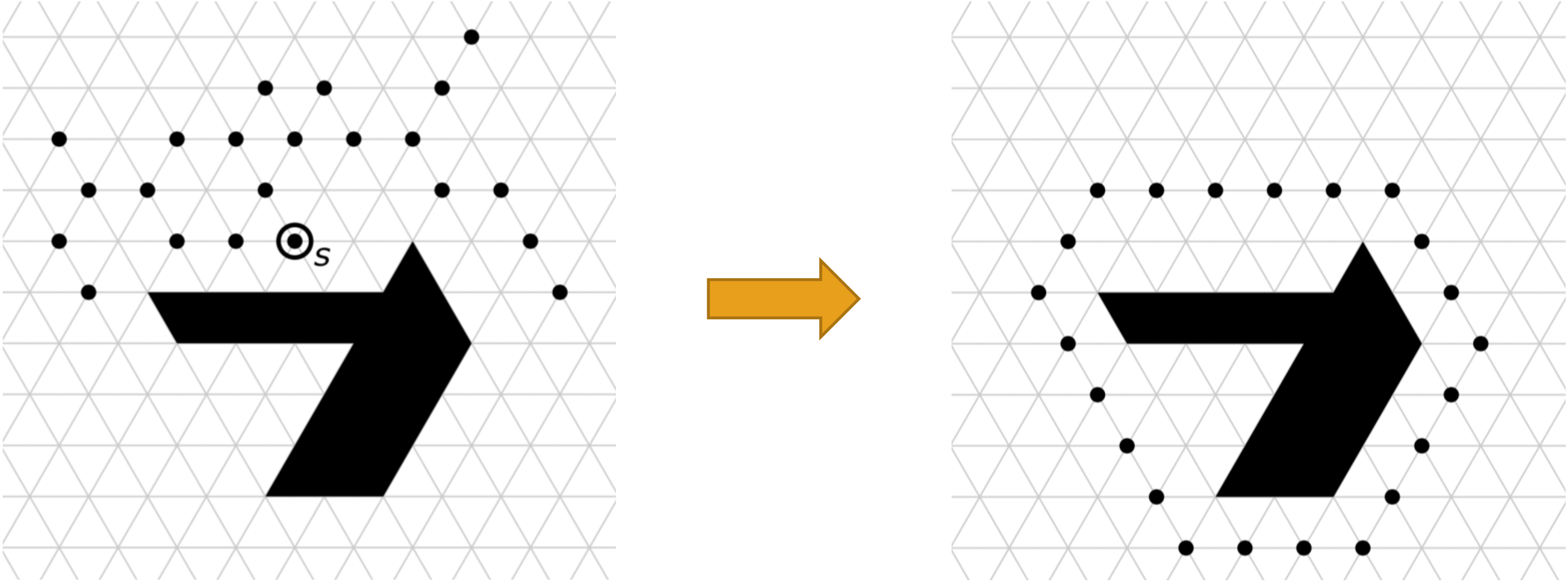
Convex Hull



Our Goal

Given: a connected object O with no holes, a connected particle system P such that $|P| \geq |C(O)|$, and a unique seed particle s which is adjacent to O.

Goal: reconfigure P so that every node of $C(O)$ is occupied by a contracted particle.



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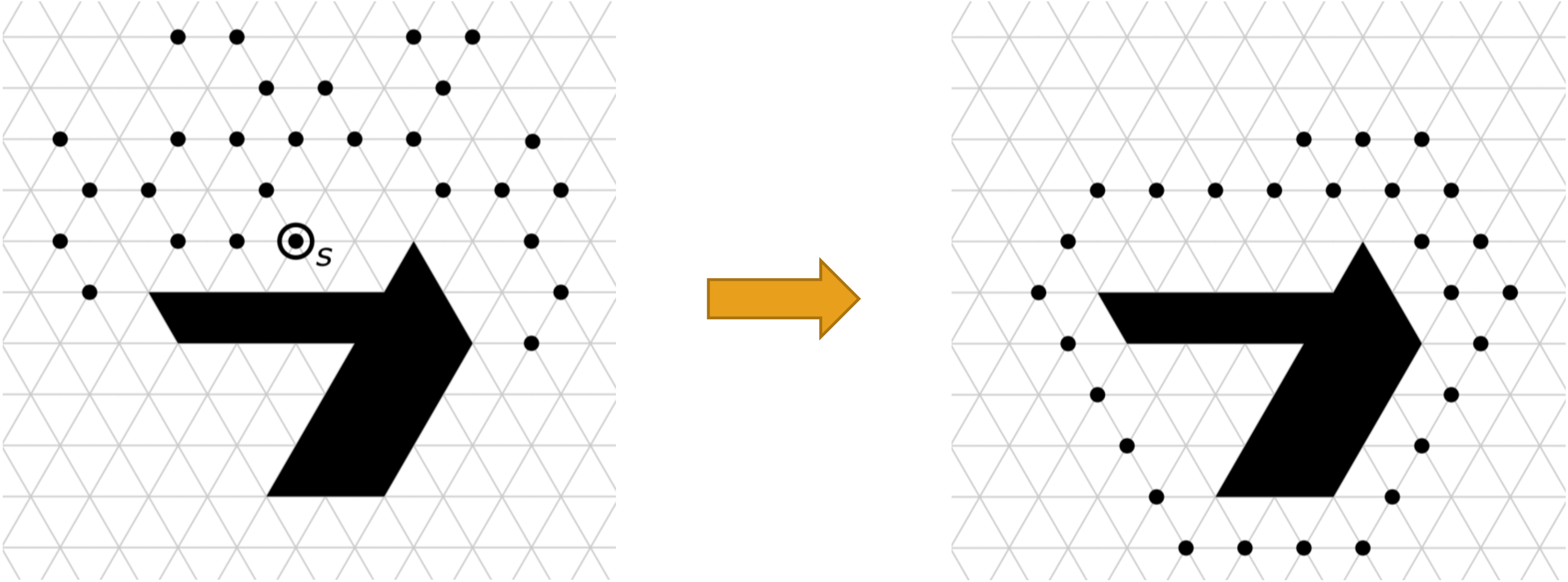

Algorithm: High Level

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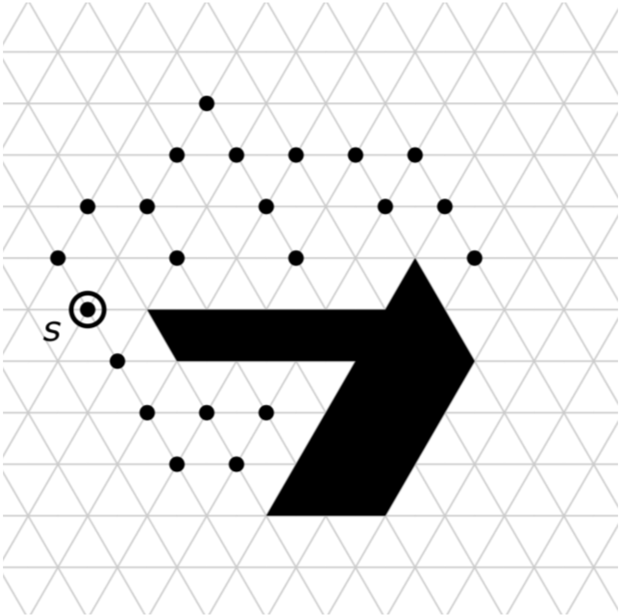

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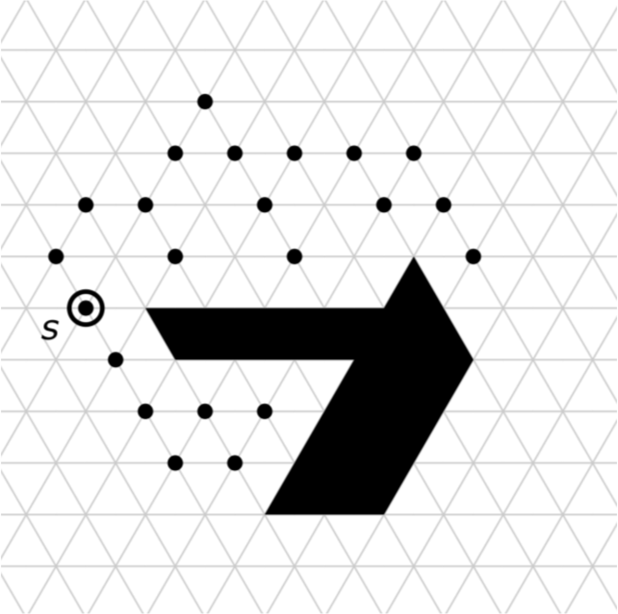
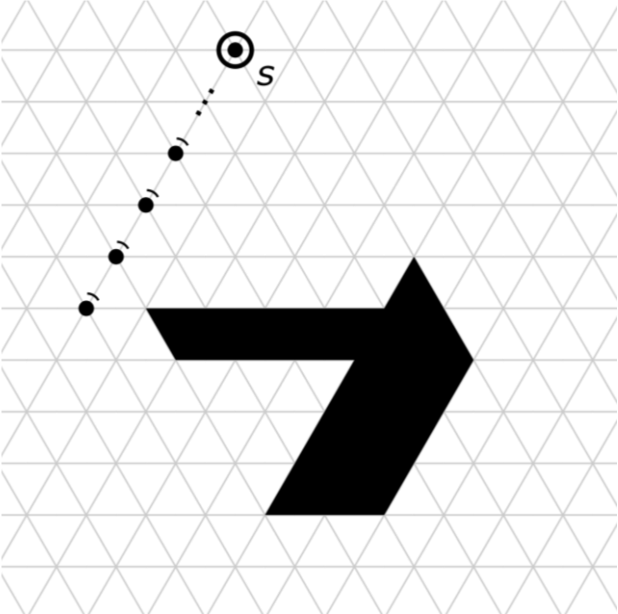
1. Phase I: Escaping the Object





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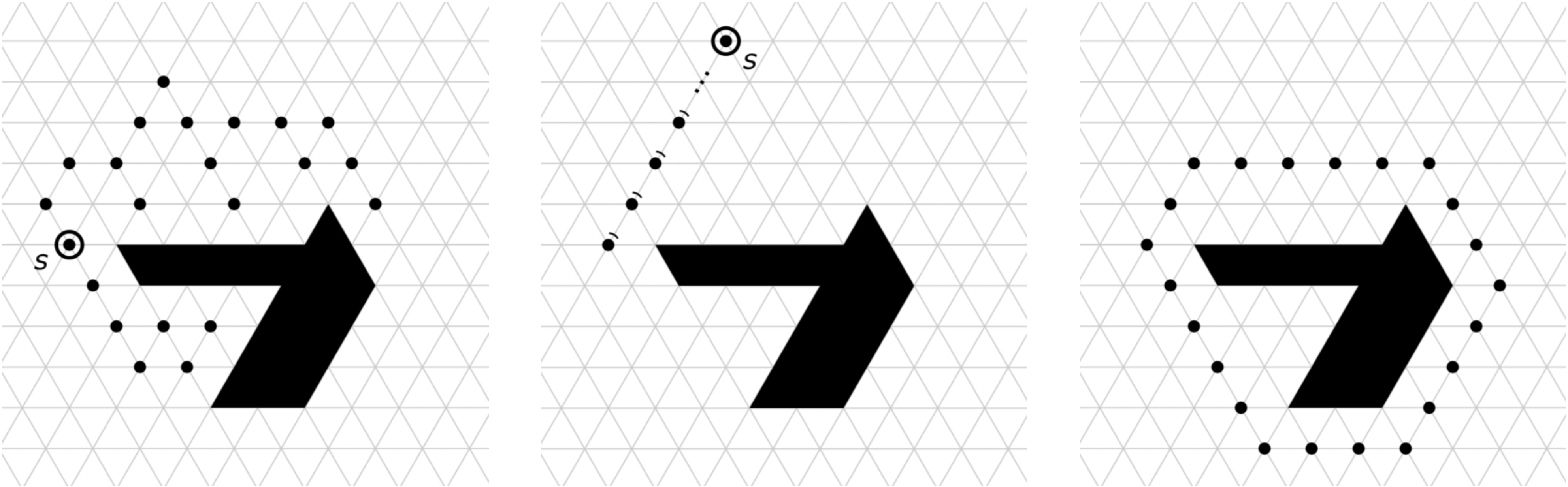

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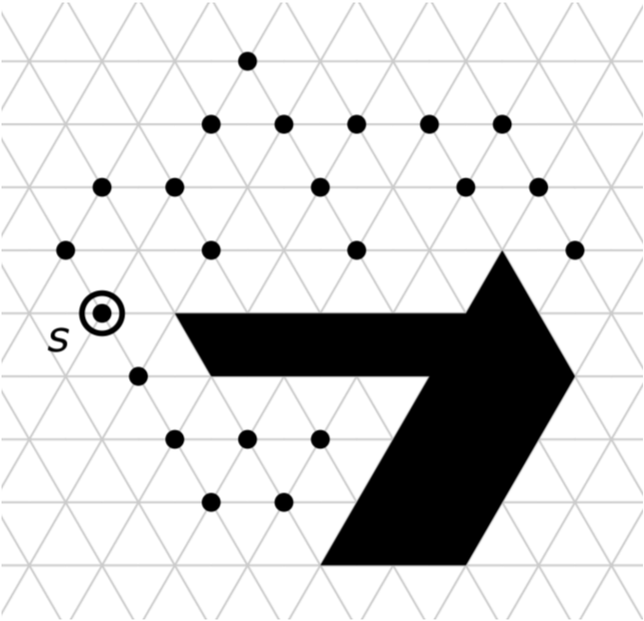

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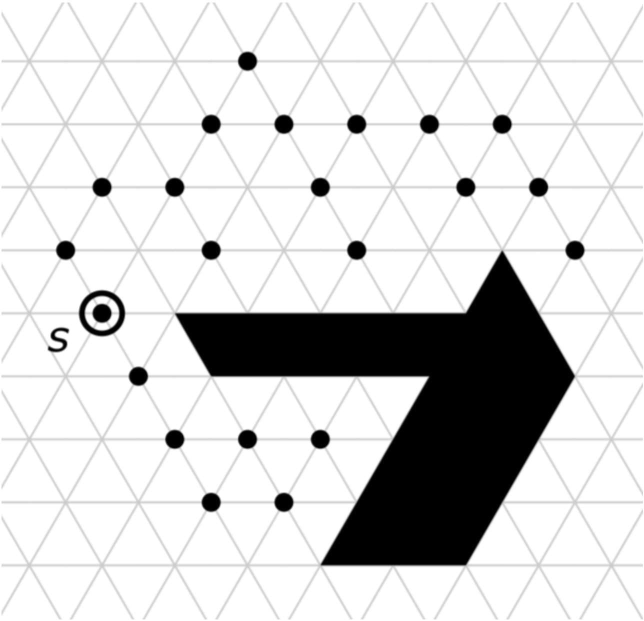

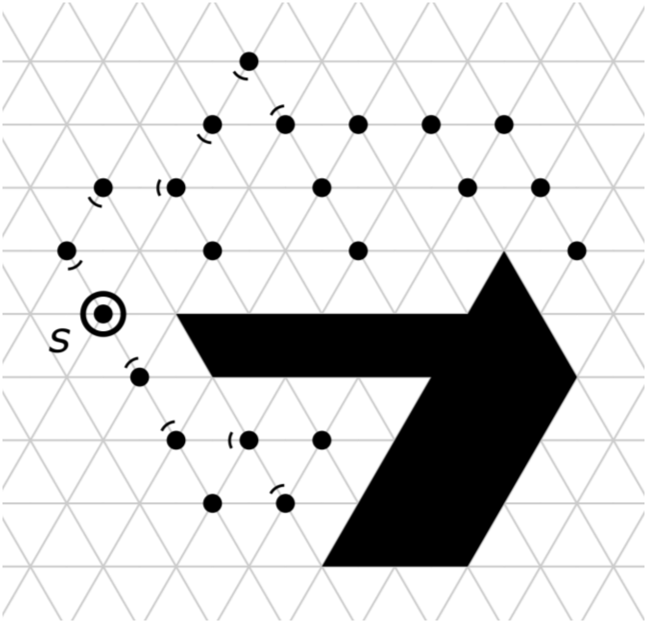

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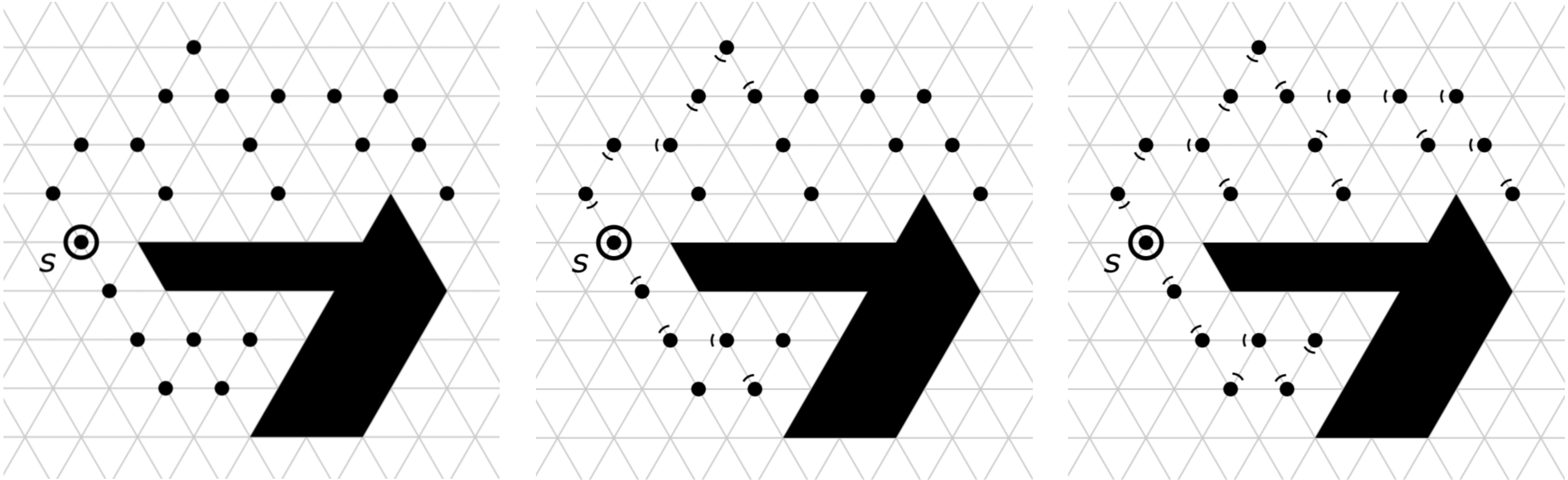

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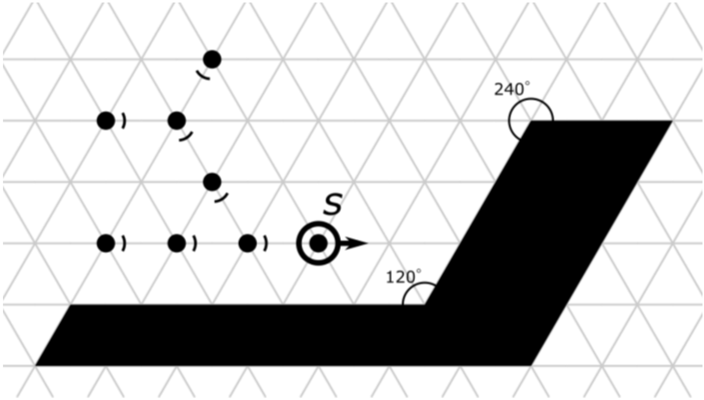

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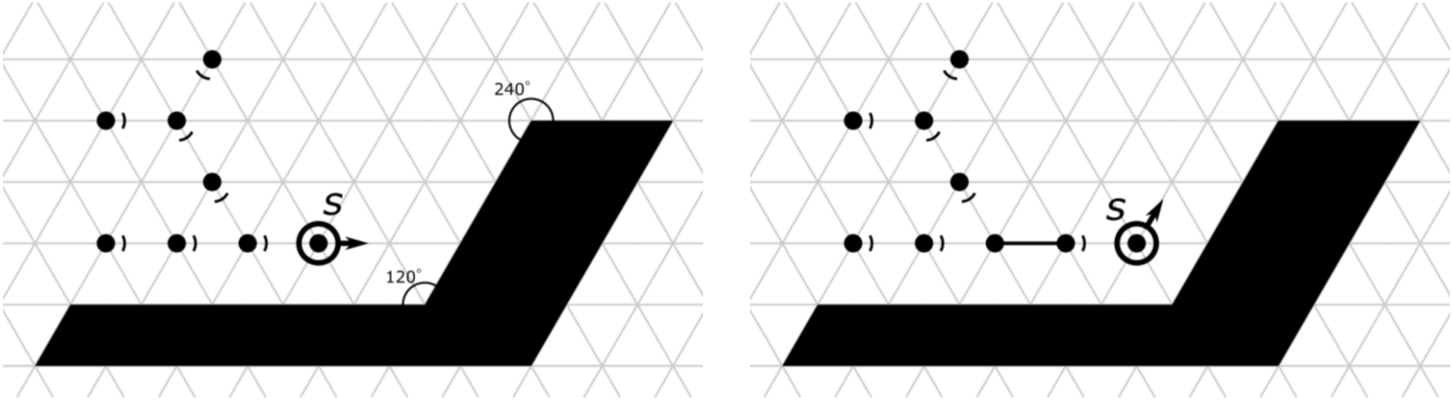

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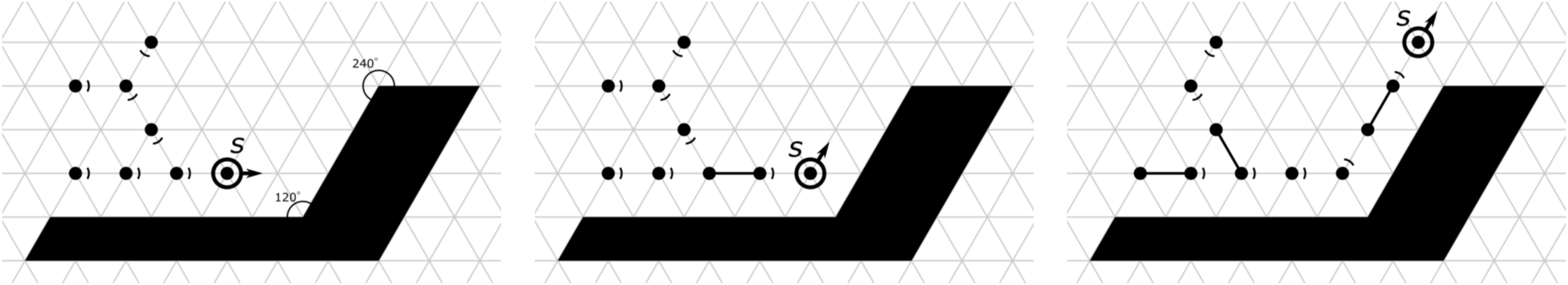

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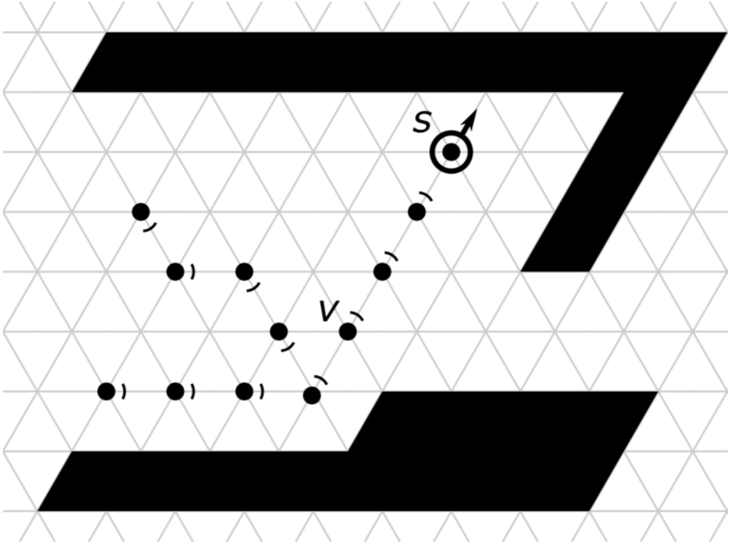

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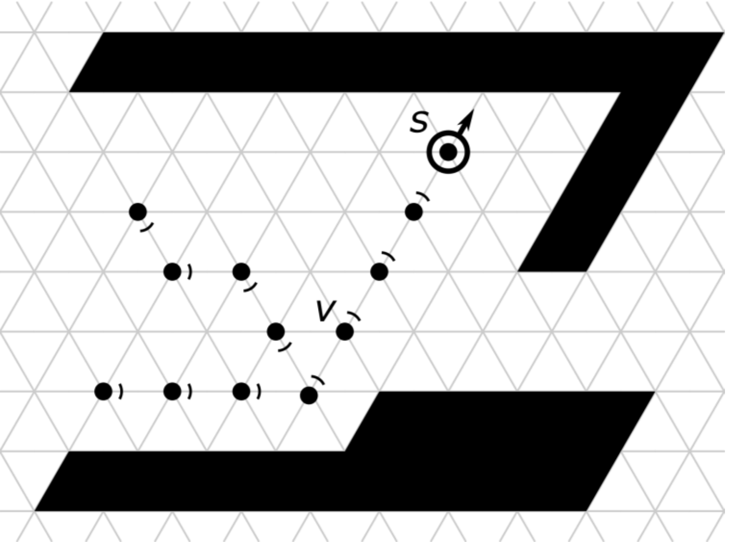

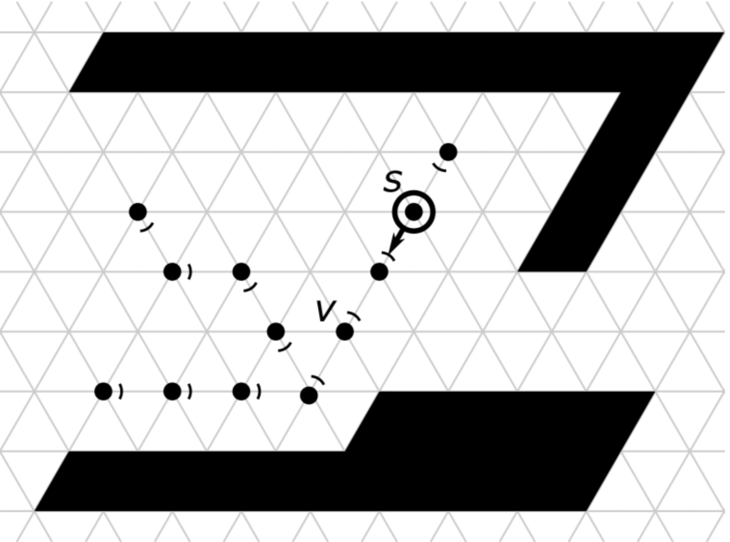

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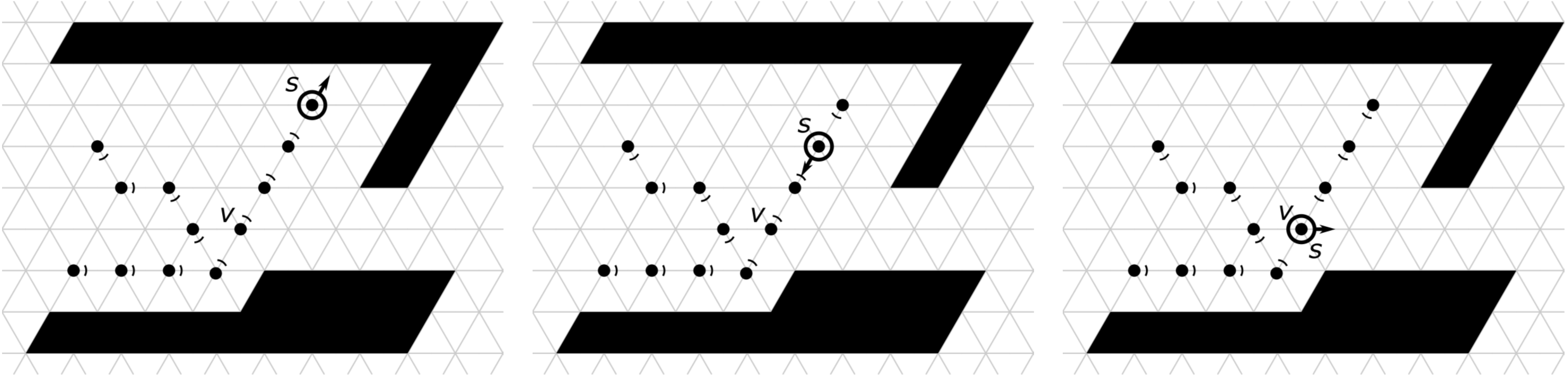

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Bending a straight line by some angle is easy in a synchronous setting, but we have asynchronous activations.

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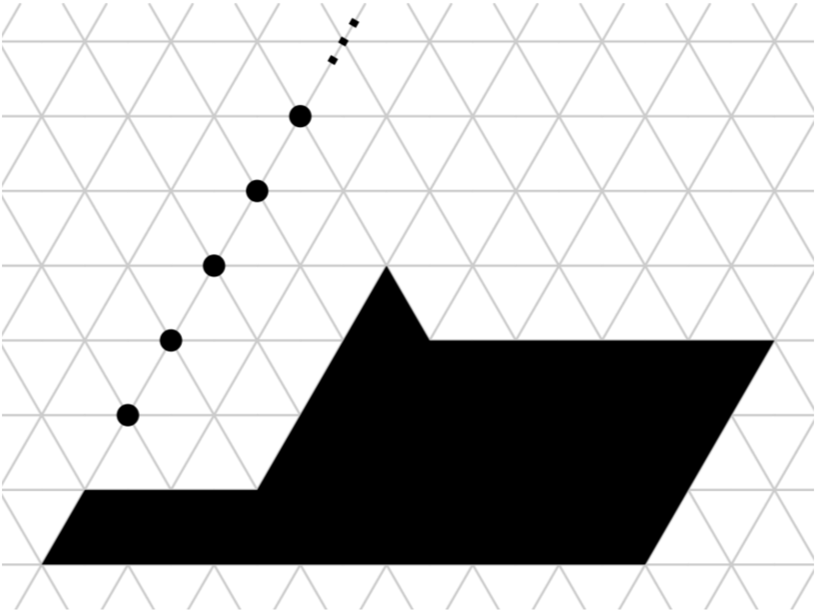

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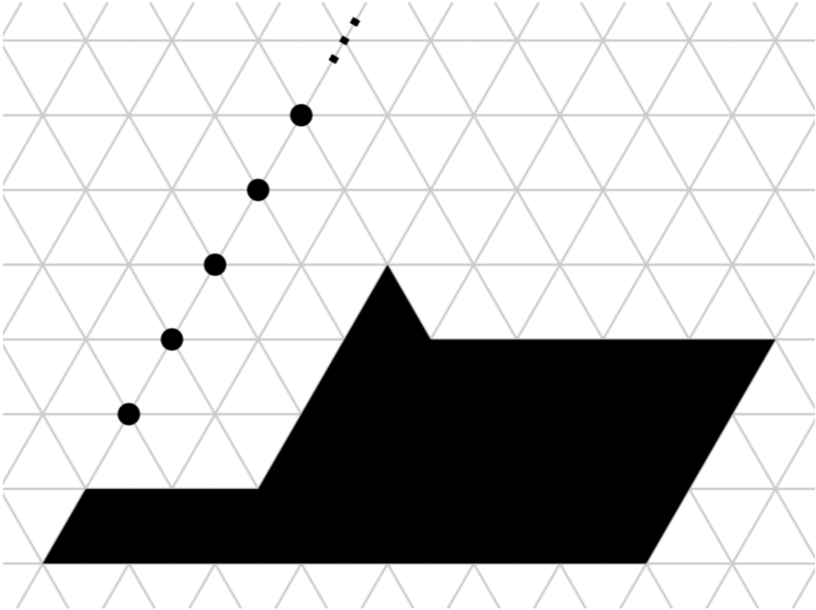

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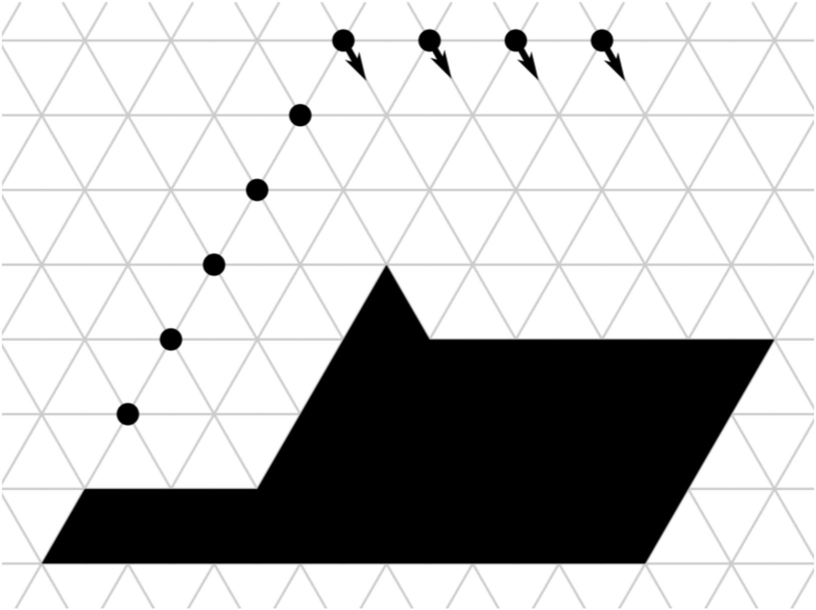

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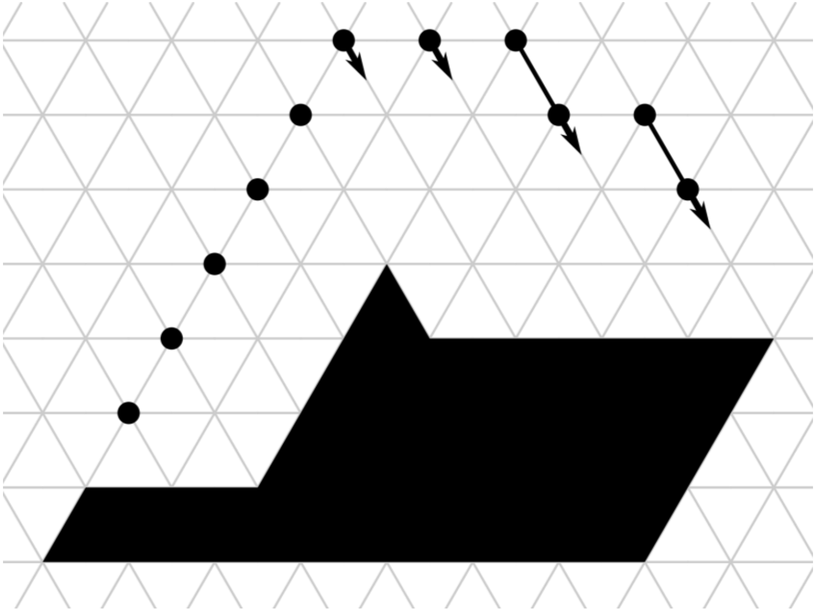

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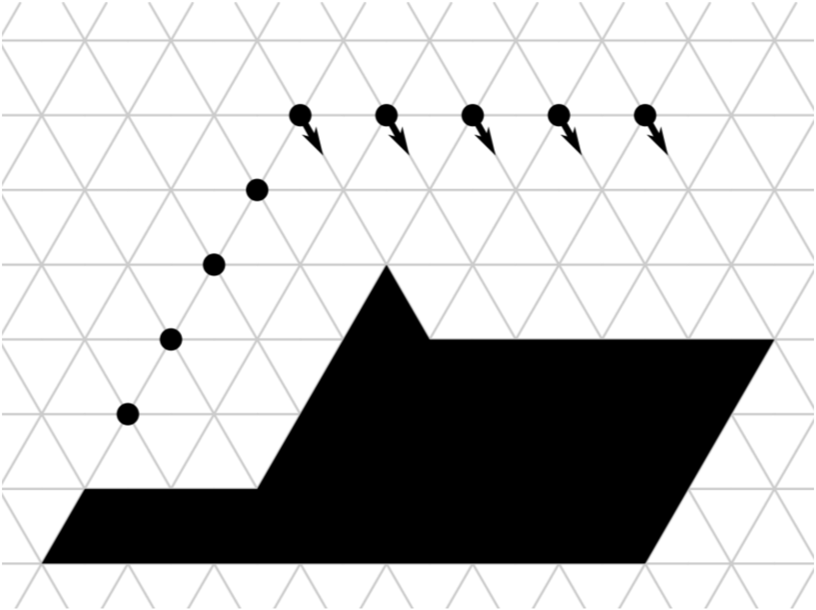

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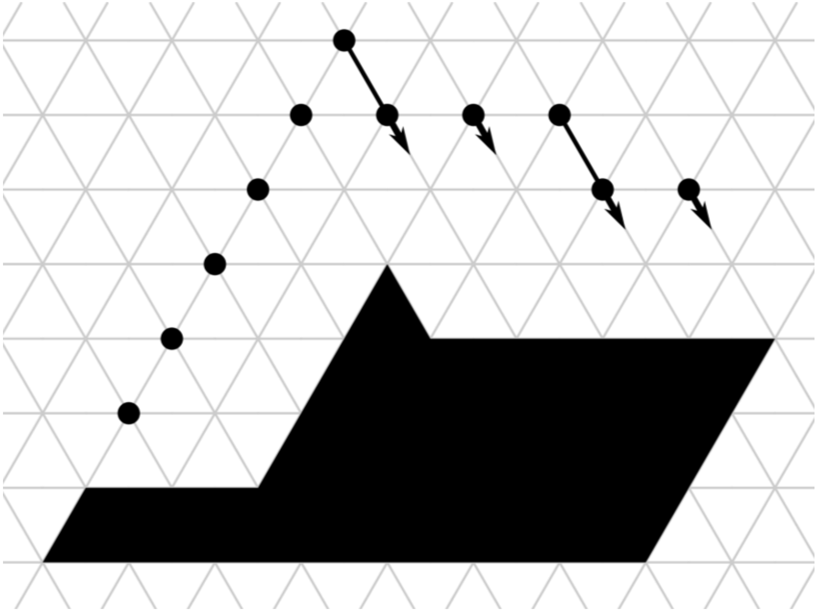

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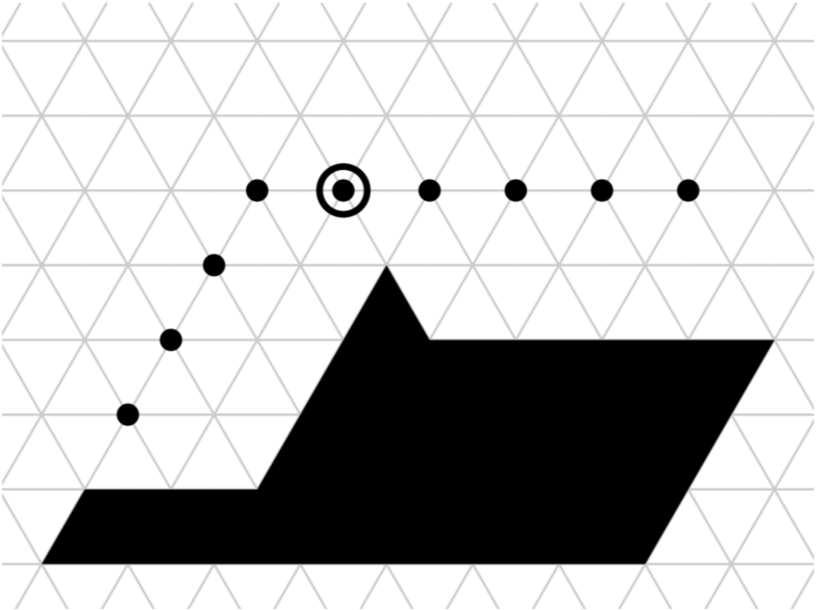

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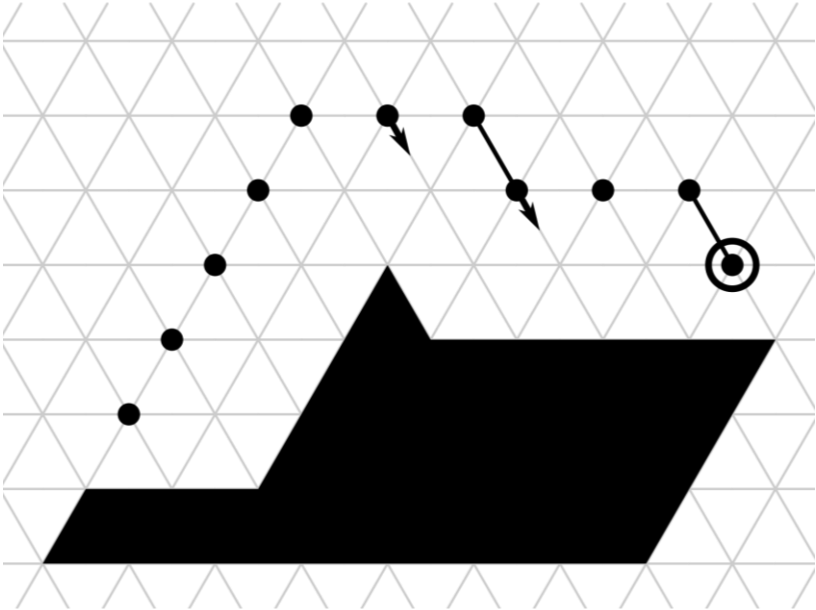

Phase II: Constructing the Convex Hull

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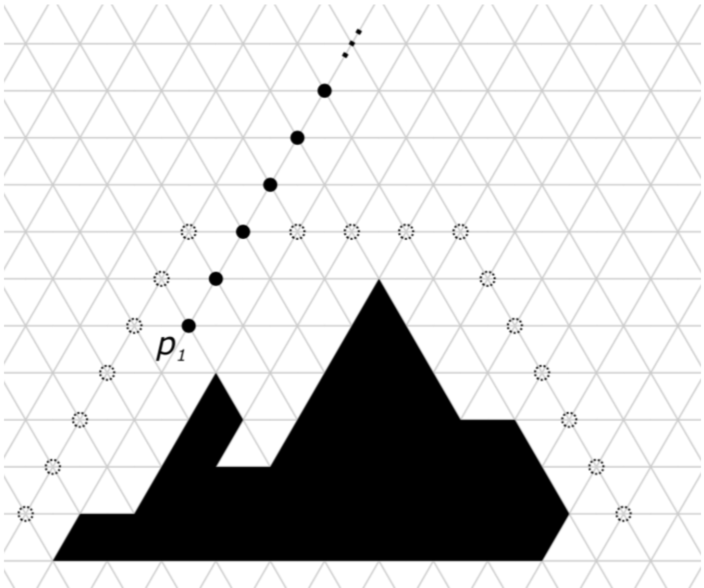

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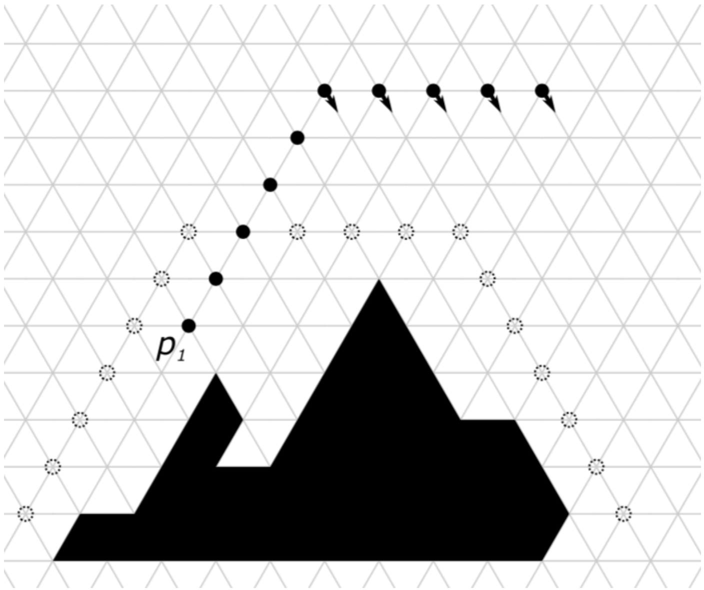

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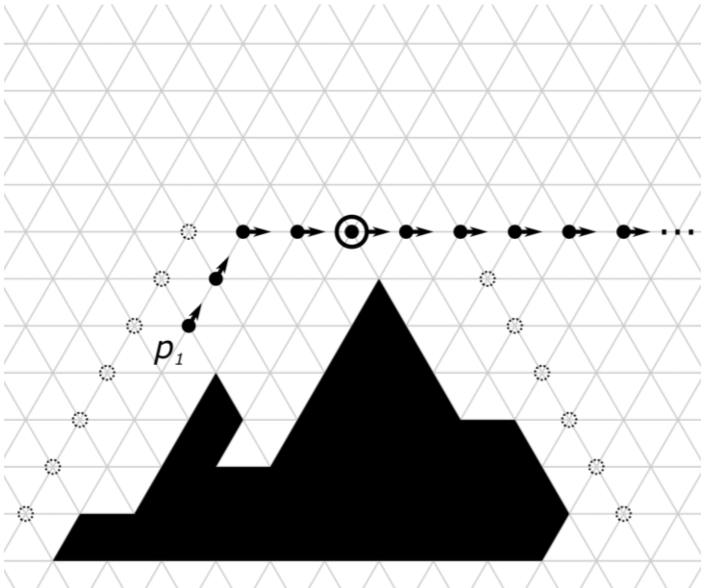

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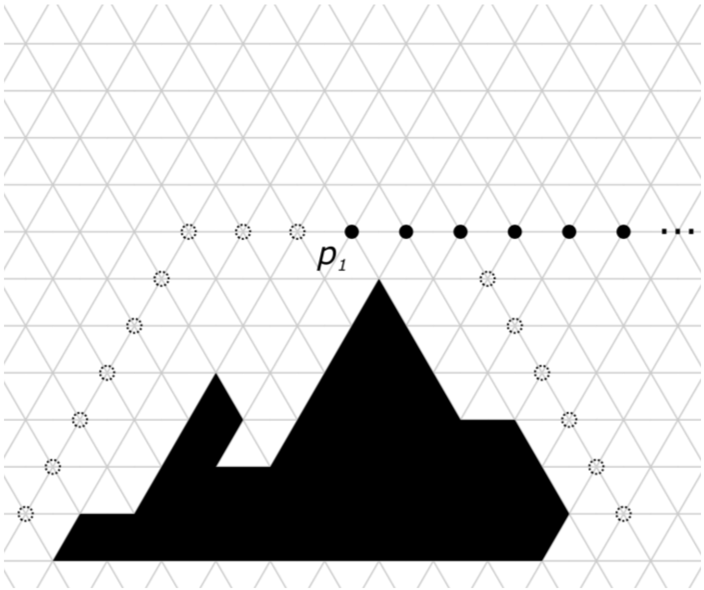

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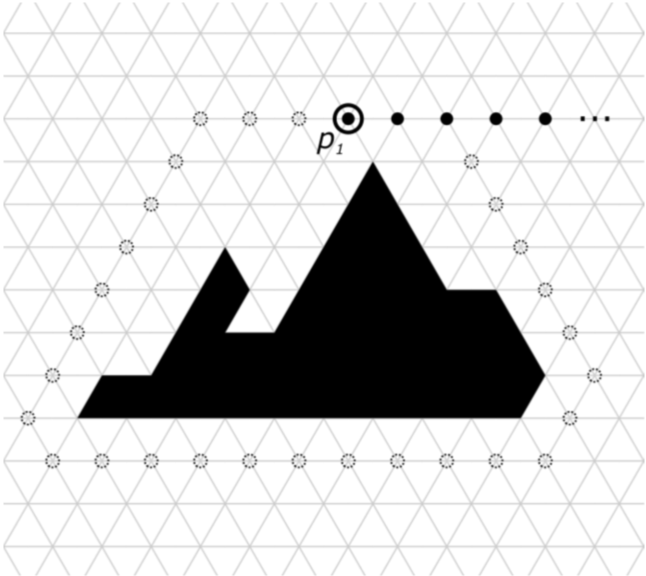

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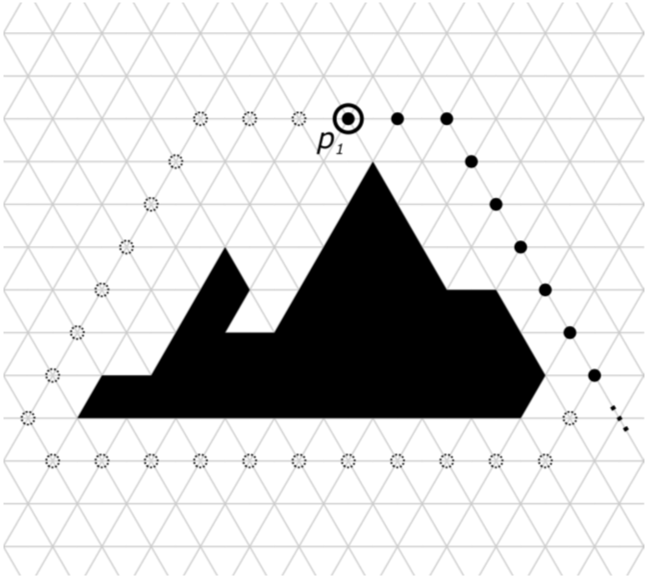

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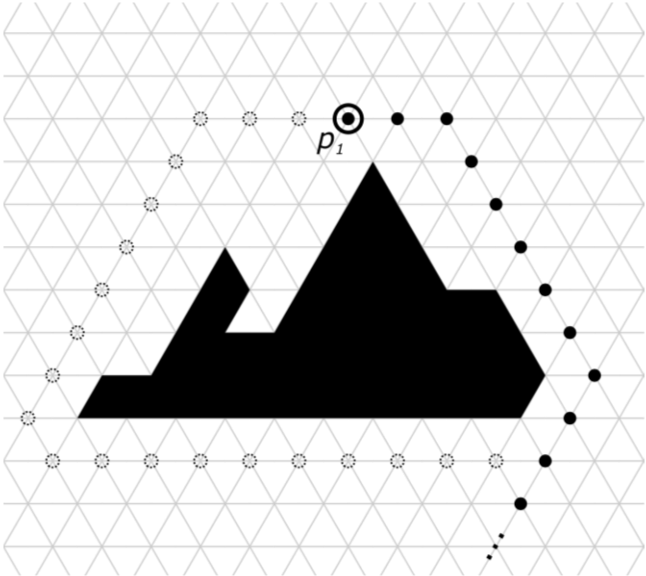

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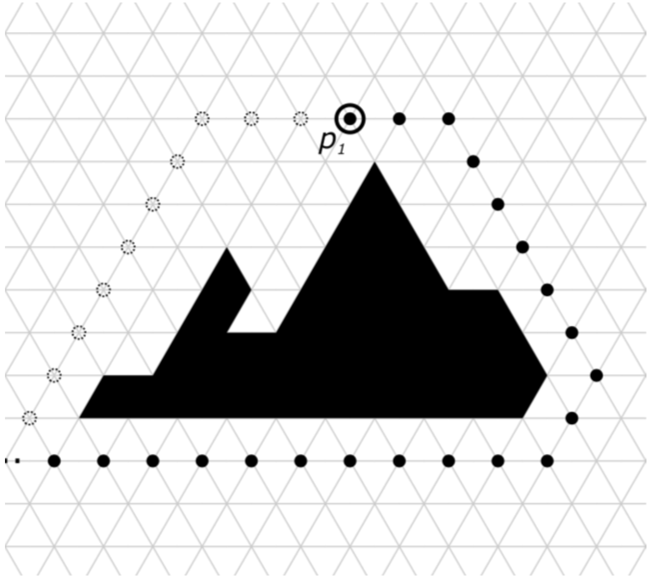

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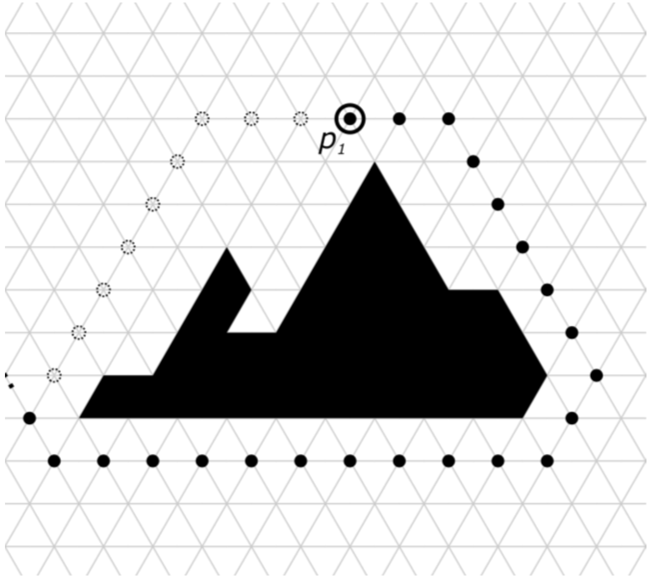

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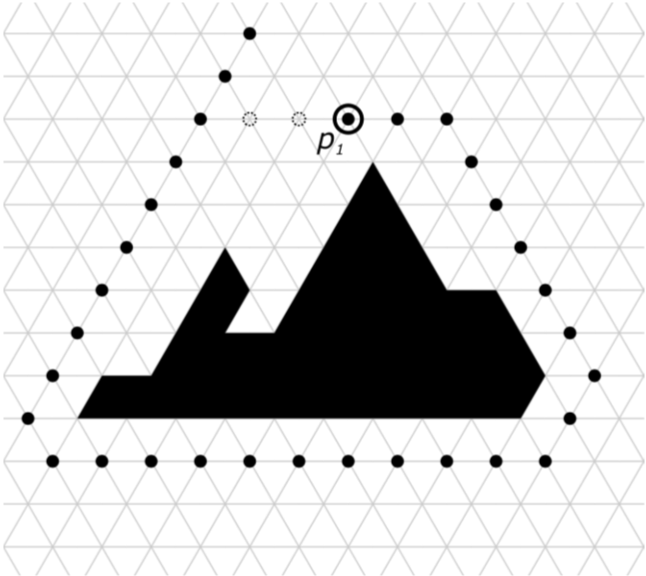

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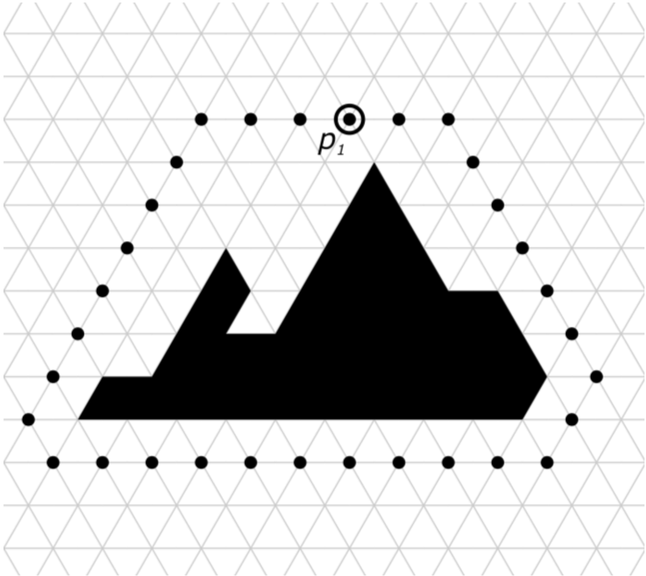


Preliminary Worst-Case Runtime Analysis

Let $n = |P|$ and m be the area occupied by O.

We measure runtime in *asynchronous rounds*.

Phase I: $\mathbf{O}(n + m)$ rounds. **?**

- Spanning forest primitive: $\mathbf{O}(n)$ rounds. ✓
- Wall following subphase: $\mathbf{O}(m)$ rounds. ✓
- Line probing subphase: $\mathbf{O}(m)$ rounds. **?**

Phase II: $\mathbf{O}(n)$ rounds. **?**

- Each line bending: $\mathbf{O}(n)$ rounds. **?**
- Move the root to the hull: ≤ 6 line bends. ✓
- Wrap the rest of the line: 6 line bends. ✓

All together: $\mathbf{O}(n + m)$ rounds…?

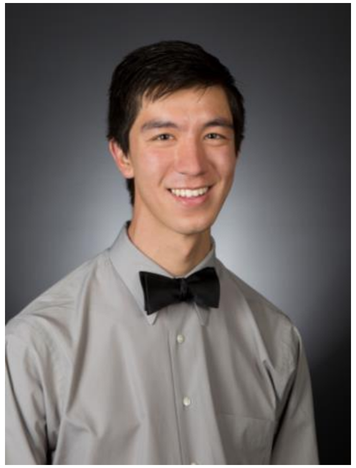
Future Work

- For convex hull formation (work-in-progress):
  - Formalize the ideas outlined here into a fully developed distributed algorithm.
  - Theoretical results: work out the details of correctness and runtime proofs.
- For Self-Organizing Particle Systems in general:
  - Pushing towards applications: bridging/filling gaps, etc.
  - Investigate more fault tolerant algorithms.
  - Generalize the existing model and algorithms to 3-dimensional space, if possible.

Collaborators

Arizona State University

Andréa W. Richa



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Christian Scheideler

Robert Gmyr

Thim Strothmann

Thank you!

sops.engineering.asu.edu