Markov Transitions between Attractor States in a Recurrent Neural Network

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Introduction

Why stochastic transitions?

Probabilistic models of cognition (see e.g. [1])

- Enable stochastic **simulations**, such as tracking noisy movements of objects [2] or reasoning with uncertainty about sequences of events [3].
- Allow for **statistical inference** using MCMC methods.
- Model observed processes in birdsong etc. [4]

In a **Markov chain,** the future depends only on the present; this captures many natural applications.

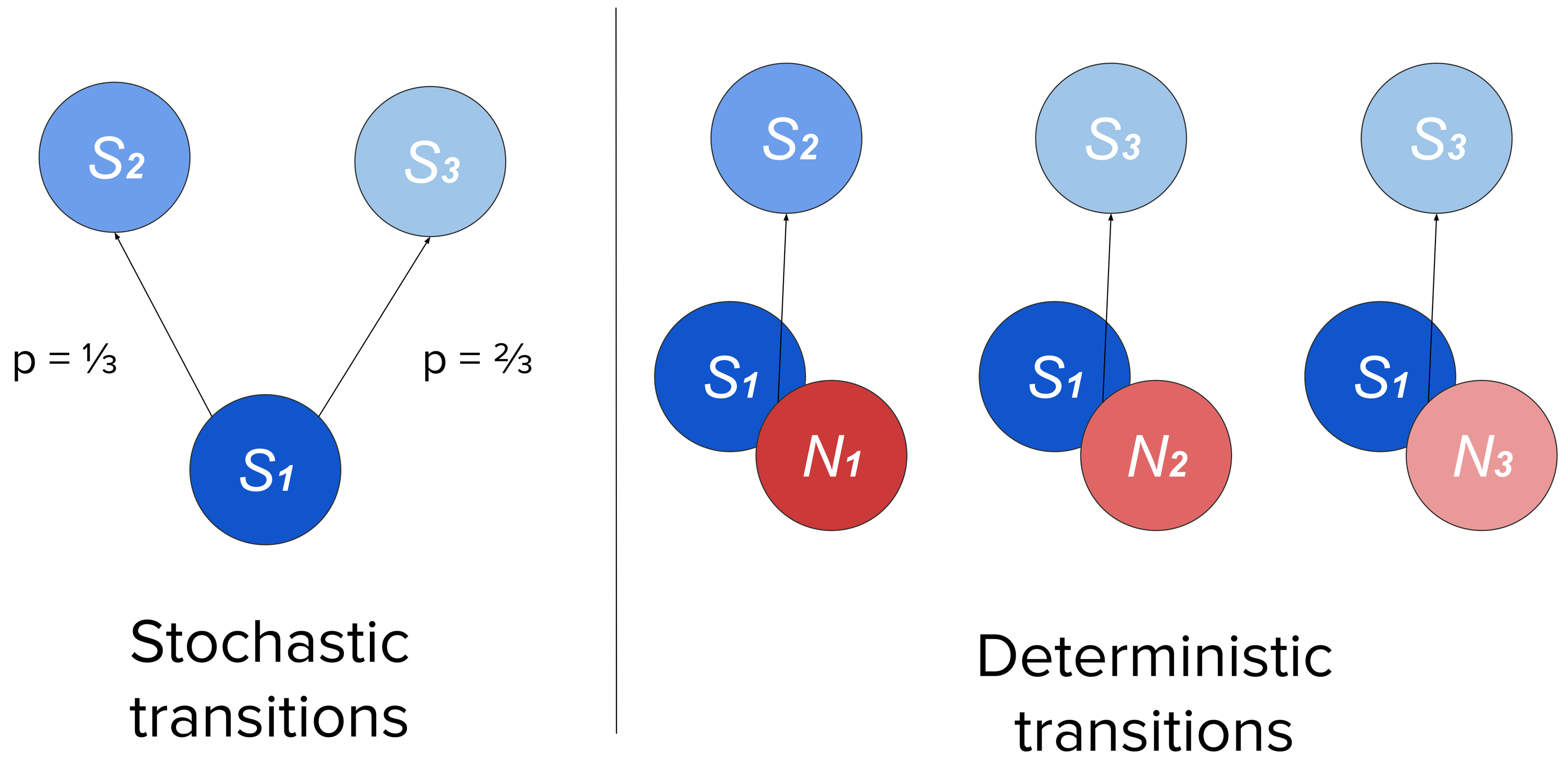
Why attractor networks?

- A **Hopfield network** [5] is a fully connected network of N neurons with update rule:

$$x_i \leftarrow \text{sign}\left(\sum_{j=1}^{n} J_{ij} x_j\right)$$

- For appropriate J_{ij}, the network state will fall into one of several chosen **attractor states, robust to noise**.
- Attractor states are a common model for memories.

How to go from deterministic to stochastic?



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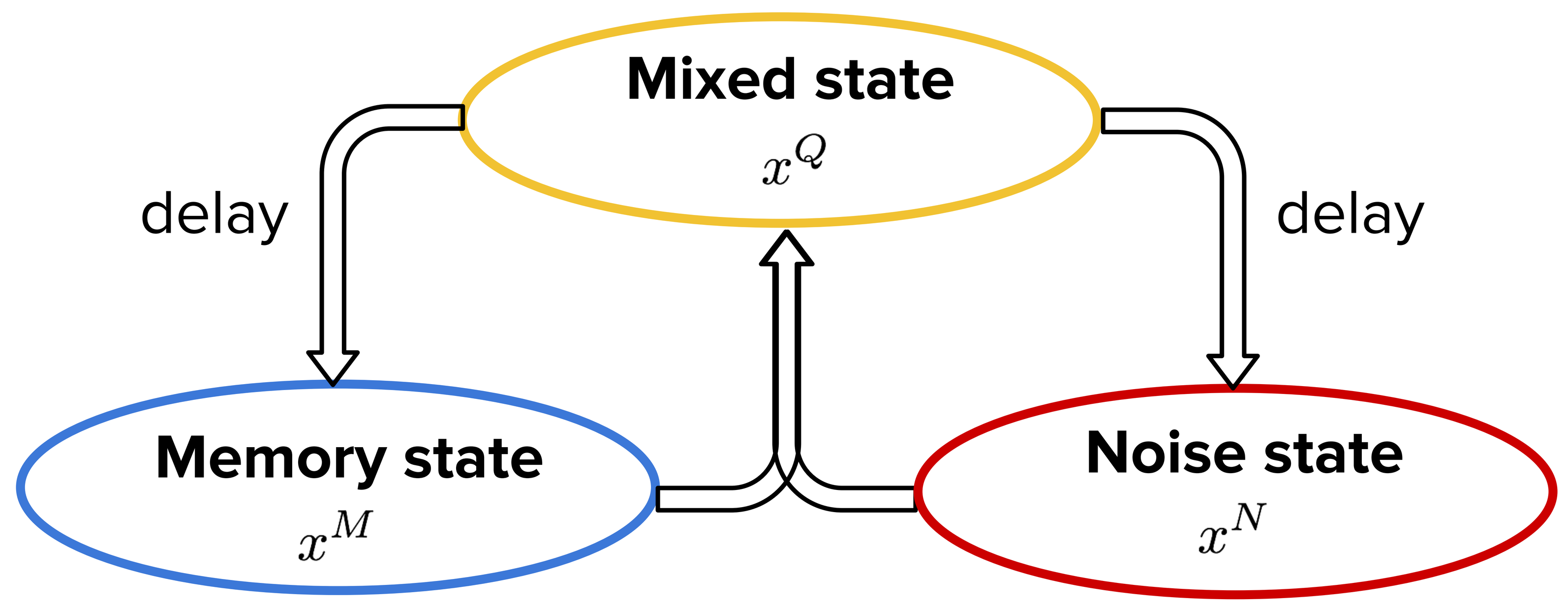
Network design

Structure of the network



$$x_i^M \leftarrow \text{sign}\left(\sum_{\ell=1}^{n_M} J_{i\ell}^M x_\ell^M + \delta_\tau^{\text{mod}}(t)\sum_{k=1}^{n_Q} J_{ik}^{MQ} x_k^Q(t-\tau)\right)$$

$$x_j^N \leftarrow \text{sign}\left(\sum_{\ell=1}^{n_N} J_{j\ell}^N x_\ell^N + \nu(t,\tau)\right)$$

$$x_k^Q \leftarrow \text{sign}\left(\sum_{\ell=1}^{n_Q} J_{k\ell}^Q x_\ell^Q + \sum_{i=1}^{n_M} J_{ki}^{QM} x_i^M + \sum_{j=1}^{n_N} J_{kj}^{QN} x_j^N\right)$$

- The function $\nu(t,\tau)$ denotes a noise function that is resampled uniformly at random at intervals of τ.
- The function $\delta_\tau^{\text{mod}}(t)$ is 1 (else 0) if $t \equiv 0 \pmod{\tau}$.

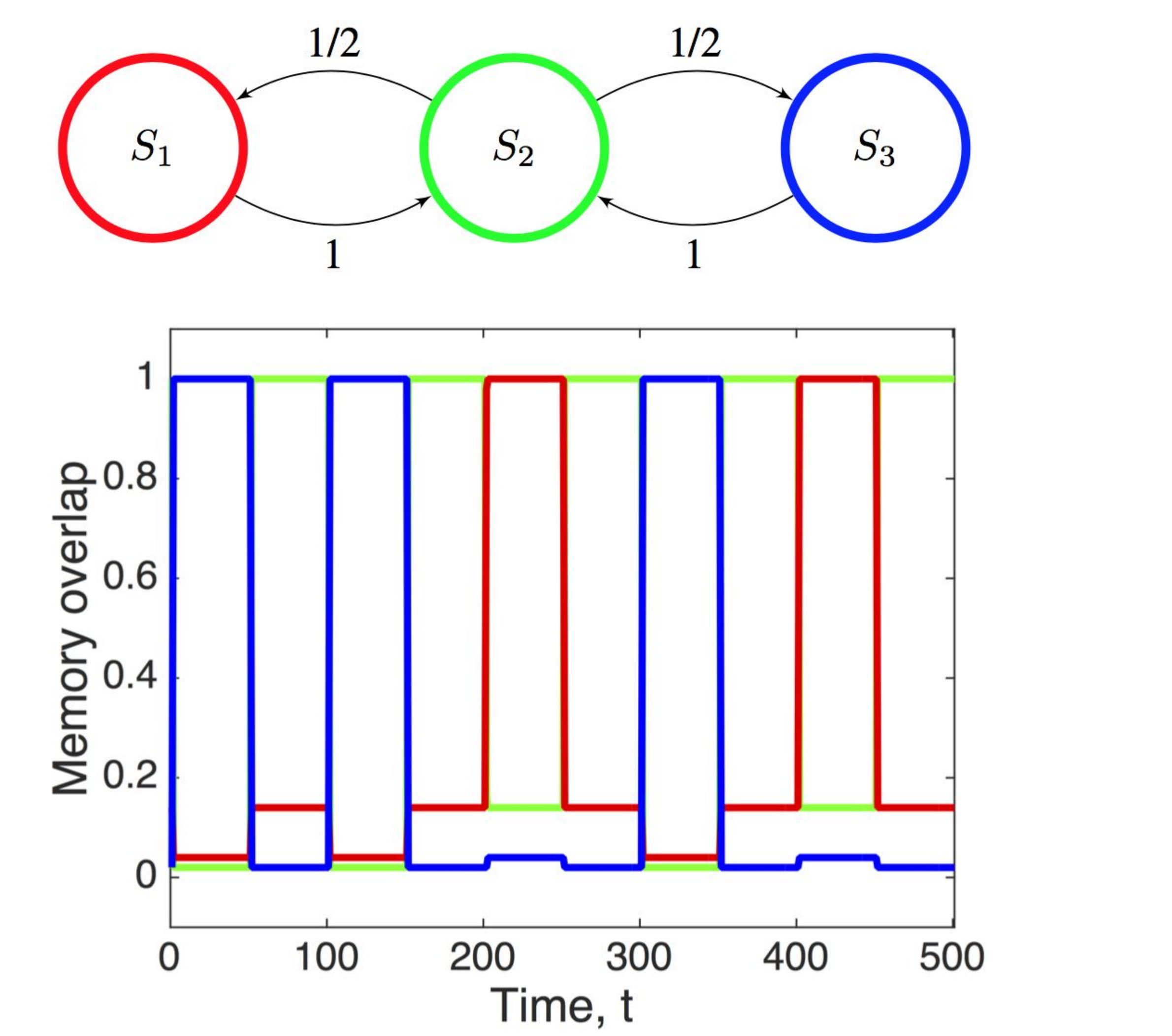
Why the mixed state?

- The next memory state is determined by the **concatenation** of the current state and a noise state. (memory(t), noise(t)) → memory(t+1)
- There is **strong linear dependence** between such concatenations, so Cover's theorem breaks down [6].
- We use a **random matrix** to project (memory, noise) pairs to a higher-dimensional space, where the corresponding mixed states are linearly separable.

Structure of noise states

- The noise state network is a **ring attractor**, with symmetric attractor states. When resampled, it falls into an attractor state **uniformly at random**.
- Attractor states ensure the network is robust to noise, compared with an ensemble of random neurons.

Example output



Learning rules

- The weight matrix J^M is learned using **Hebb's rule**, J^Q using the **perceptron rule**, and J^{MQ} as in [7].
- The weight matrices J^{QM} and J^{QN} are random, while J^N is chosen to instantiate a ring attractor network.

Future directions

- Determine network **capacity** and **noise robustness**.
- Construct statistical inference model for attractor networks using **Markov Chain Monte Carlo** methods.
- Improve **clock gating mechanism** for transitions.
- Develop biologically plausible **on-line learning rule**.

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